APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY A
FIRST SEMESTER M.TECH DEGREE EXAMINATION

ELECTRICAL AND ELECTRONICS ENGINEERING

04 MA 6301: Advanced Mathematics

Duration: 3 Hours Maximum Marks: 60

Part A

(Answer **all** questions. Each question carries 3 marks.)

1. Evaluate $\int_C \frac{e^z}{(z+i)^3}\, dz$ where C is the circle $|z| = 2$.
2. Prove that the set {(1,1,4), (2,1,3), (0,1,6)} is a basis for R^3.
3. Find the value of k so that the following is the probability distribution of a discrete random variable.

x:	0	1	2	3	4	5	6	7	8
P(x):	k	3k	5k	7k	9k	11k	13k	15k	17k

Also find the distribution function of X.

4. Explain Auto-Correlation function.
5. Why is Powell's method called a pattern search method?
6. Explain the term conjugate directions.
7. Minimize $f(x_1,x_2)= 6x_1^2+2x_2^2-6x_1x_2-x_1-2x_2$ by taking the starting point as $X_1=\binom{0}{0}$ using Newton's method.
8. Define the following a) Hessian matrix of a function. b) Gradient of a function.

Part B

(Each full question carries 6 marks)

9. State and prove Poisson's integral formula for a circle.

OR

10. The bounding diameter of a semi-circular plate of radius 'a' cm is kept at 0^0 C and the temperature along the semi-circular boundary is given by

$$u(a, \theta) = \begin{cases} 50\theta, \text{ when } 0 < \theta \le \pi/2 \\ 50(\pi-\theta), \text{ when } \frac{\pi}{2} < \theta < \pi \end{cases}$$

Find the steady state temperature function $u(r,\theta)$.

11. a) Define the following
 i) Linear transformation in vector spaces ii) Isomorphism of vector spaces.
 b) Prove that any two bases in a vector space contains the same number of elements.

OR

PTO

12. Let V be the set of polynomials $a_0 + a_1t + a_2t^2 + ... + a_nt^n$ with coefficients a_i from a field K. Show that V is a vector space under usual operations of addition of polynomials and multiplication by a constant.

13. The distribution function of a continuous random variable is given by

$$F(x) = \begin{cases} x^2, & 0 \le x < 1/2 \\ 1 - \frac{3}{25}(3-x)^2, & 1/2 \le x < 3 \\ 1, & x \ge 3. \end{cases}$$

Find the pdf of X. Evaluate $P(|X| \le 1)$ and $P(1/3 \le X \le 4)$.

OR

14. A random variable X has the pdf $f(x) = \begin{cases} \frac{k}{1+x^2}, & -\infty < x < \infty \\ 0, & \text{otherwise.} \end{cases}$

Determine the value of 'k' and find the distribution function of X. Also evaluate $P(X \ge 0)$.

15. a) A man tosses a fair coin until 3 heads occur in a row. Let X_n denotes the longest string of heads ending at the n^{th} trial. Find the transition probability matrix.

b) Define a random process and classify it.

OR

16. a) Derive Chapman Kolmogorov equation b) Let $\{X_n : n \ge 0\}$ be a Markov chain with three states 0,1,2 and with transition matrix $\begin{pmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{pmatrix}$ with initial distribution $P(X_0 = i) = 1/3$, i=0,1,2.

Find $P(X_2 = 2, X_1 = 1 / X_0 = 2)$.

17. Explain Hooke-Jeeves' method of unconstrained optimization.

OR

18. Using Powell's method find the minimum of the function $f = 4x_1^2 + 3x_2^2 - 5x_1x_2 - 8x_1$ starting from the point (0,0).

19. Minimize $f = 2x_1^2 + x_2^2$ using steepest descent method with the starting point (1,2).

OR

20. Find the maximum of the function $2x_1 + x_2 + 10$ subject to $x_1 + 2x_2^2 = 3$ using Lagrange multiplier method.