Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Third Semester B.Tech Degree Examination December 2020 (2019 Scheme)

Course Code: MAT201

Course Name: Partial Differential equations and Complex analysis

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions. Each question carries 3 marks Marks

1	Form the PDE for the equation $z = f(x^2 - y^2)$ where f is an arbitrary function.	(3)
2	Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$ given that when $x = 0, z = e^y$ and $\frac{\partial z}{\partial x} = 1$	(3)
3	Write the conditions in which a tightly stretched string of length l with fixed end is initially in equilibrium position and is set vibrating by giving each point a velocity $v_0 sin^3 \frac{\pi x}{l}$	(3)
4	Write down the possible solutions of one dimensional heat equation.	(3)
5	Find the real part and imaginary part of the function $f(z) = 5z^2 - 12z + 3 + 2i$ and find their values at $z = 4 - 3i$	(3)
6	Find the fixed points of the mapping $w = (a + ib)z^2$	(3)
7	Evaluate $\oint_C \frac{e^z}{z-2} dz$ where C is $\|z\| = 3$	(3)
8	Find the Maclaurin series expansion of $\int_0^z e^{-t^2}\, dt$	(3)
9	Find the Laurent series of $z^{-5} sinz$ with centre 0	(3)
10	Find the residue at poles for the function $f(z) = \frac{-8}{1+z^2}$	(3)

PART B

Answer any one full question from each module. Each question carries 14 marks

Module 1

11(a)	Solve $(x^2 - y^2 - z^2)p + 2xyq = 2xz$	(7)
(b)	Solve $(p^2 + q^2)y = qz$ by Charpit's method	(7)
12(a)	Find the differential equation of all planes which are at a constant distance 'a' from the origin	(7)
(b)	Solve $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$ where $u(x, 0) = 4e^{-x}$ by the method of separation of variables.	(7)

Module 2

13 The points of trisection of a string are pulled aside through the same distance on opposite sides of the position of equilibrium and the string is released from rest. Derive an expression for the displacement of the string at subsequent time. (14)

14 Solve the boundary value problem $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}, 0 < x < l, \frac{\partial u}{\partial x}(0,t) = 0, \frac{\partial u}{\partial x}(l,t) = 0, u(x,0) = x$ (14)

Module 3

15(a) Determine 'a' so that the function $u = e^{-\pi x} cosay$ is harmonic and find its harmonic conjugate. (7)

(b) Is the function $f(z) = \begin{cases} \frac{Rez^2}{|z|^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ continuous or not at $z = 0$ (7)

16(a) Find the image of the region $\left|z - \frac{1}{2}\right| \leq \frac{1}{2}$ under the transformation $w = \frac{1}{z}$ (7)

(b) Show that $f(z) = |z|^2$ is differentiable only at $z = 0$, hence it is nowhere analytic. (7)

Module 4

17(a) Evaluate $\oint_C \frac{z^3-6}{2z-i} dz$ where C is $|z| = 1$ in counterclockwise direction (7)

(b) Find the Maclaurin series expansion of $\frac{z+2}{1-z^2}$ (7)

18(a) Integrate $\oint_C \frac{sinh2z}{(z-\frac{1}{2})^4} dz$ in counterclockwise direction around the unit circle (7)

(b) Find the Taylor series expansion of $f(z) = \frac{1}{1+z}$ with centre $z_0 = -i$ (7)

Module 5

19(a) Find the Laurents series of that $f(z) = \frac{e^z}{(z-1)^2}$ that converge for $0 < |z-1| < R$ and determine the region of convergence (7)

(b) Evaluate $\oint_0^{2\pi} \frac{sin^2\theta}{5-4cos\theta} \, d\theta$ (7)

20(a) Evaluate $\oint_C tan2\pi z \, dz$ counter clockwise around C :$|z - 0.2| = 0.2$ (7)

(b) Find the principal value of $\int_{-\infty}^{\infty} \frac{dx}{(x^2-3x+2)(x^2+1)}$ (7)

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