Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Third Semester B.Tech Degree (S,FE) Examination December 2020

Course Code: CS201

Course Name: DISCRETE COMPUTATIONAL STRUCTURES

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks. Marks

1		Draw the Hasse diagram of posets under the partial order relation of divisibility A={1,2,3,5,6,10,15,30}	(3)
2		Determine whether the relation $R=\{(a,b) \mid a \geq b\}$ on the set of real numbers is an equivalence relation.	(3)
3		In how many ways can letters in the English alphabet be arranged so that there are exactly 7 letters between the letters 'a' and 'b'.	(3)
4		Among the first 500 positive integers, determine the integers which are not divisible by 2, nor by 3, nor by 5.	(3)

PART B

Answer any two full questions, each carries 9 marks.

5	a)	Let f(x)=x+2, g(x)= x-2 and h(x)=3x for $x \in R$, where R is the set of real numbers. Find gof , fog, fof, gog, foh, hog , hoh and fohog	(4)
	b)	If the function f is defined by $f(x) = x^2 + 1$ on the set {-2, -1, 0, 1, 2}, find the range of f.	(5)
6	a)	Show that the set N of natural numbers is a semigroup under the operation x*y = max(x,y). Is it a monoid?	(4)
	b)	8 scientists and 5 politicians take part in a conference. In how many ways can they be seated in a single row if (i) no 2 politician must sit together (ii) no 2 scientist must sit together.	(5)
7	a)	Solve the recurrence relation $a_n = 6\, a_{n-1} - 9\, a_{n-2}$, $n \geq 2$ and $a_o = 1$ and $a_1 = 4$	(5)
	b)	Show that A X (B∩C) = (AXB) ∩ (AXC).	(4)

PART C

Answer all questions, each carries 3 marks.

8		Define group homomorphism.	(3)
9		How many proper subgroups will be there for a group of order 11? Justify your Answer.	(3)
10		Let $(L,\leq)$ be a lattice and a,b,c,d elements of L. Prove that if $a \leq c$ and $b \leq d$ then $a \vee b \leq c \vee d$	(3)
11		Define a complemented lattice. Give an example.	(3)

PART D

Answer any two full questions, each carries 9 marks.

12	a)	What is a complete lattice? Give an example.	(4)
	b)	Show that the set of all positive rational numbers Q+ forms an abelian group under the operation * defined by a*b=(ab)/2 for a,b ϵ Q+.	(5)
13	a)	Define a Boolean algebra. Illustrate a two element Boolean Algebra with an example.	(4)
	b)	Let H = {0, 3, 6} in Z_9 under addition. What are the cosets of H in Z_9?	(5)
14	a)	Verify that the set{ 0, 1, 2, 3, 4, 5 } under addition and multiplication modulo 6 is group or not.	(4)
	b)	A= {2, 3, 4, 6, 12, 18, 24, 36} with partial order of divisibility. Determine whether the POSET is a lattice or not.	(5)

PART E

Answer any four full questions, each carries 10 marks.

15	a)	Without using truth tables prove that $\neg(P \wedge Q) \rightarrow (\neg P \vee (\neg P \vee Q)) \Leftrightarrow (\neg P \vee Q)$	(5)
	b)	Suppose x is a real number. Consider the statement "If $x^2 = 4$, then x = 2." Construct the converse, inverse, and contrapositive.	(5)
16	a)	Prove that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.	(5)
	b)	Prove that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x) P(x) \wedge (\exists x) Q(x)$.	(5)
17	a)	Show that the premises " A student in this class has not read the book " and " Everyone in this class passed the first exam " imply the conclusion "Someone who passed the first exam has not read the book".	(5)
	b)	Show that the premises, " It is not sunny this afternoon and it is colder than yesterday" , " We will go swimming only if it is sunny" , " If we do not go swimming , then we will take a canoe trip" , and " If we take a canoe trip , then we will be home by sunset " lead to the conclusion " We will be home by sunset ".	(5)
18	a)	Show that $(x)(P(x) \rightarrow Q(x)) \wedge (x)(Q(x) \rightarrow R(x)) \Rightarrow (x)(P(x) \rightarrow R(x))$	(5)
	b)	Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \rightarrow R$, $P \rightarrow M$ and $\neg M$.	(5)
19	a)	Show that $(\exists x) M(x)$ follows logically from the premises $(x)(H(x) \rightarrow M(x))$ and $(\exists x) H(x)$	(5)
	b)	Show that $((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R)$ is a tautology.	(5)
20	a)	Prove by contradiction " If 3n + 2 is an odd integer , then n is odd ".	(5)
	b)	Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$	(5)

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