Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Eighth semester B.Tech degree examinations, September 2020

Course Code: MA486

Course Name: Course Name: ADVANCED NUMERICAL TECHNIQUES

Max. Marks: 100 Duration: 3 Hours

PART A

Answer any two full questions, each carries 15 marks

1 a) Find the largest eigen value and the corresponding eigen vector of the matrix by using power method. $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ (7)

b) Solve the equations $10x + y + z = 12,\ 2x + 10y + z = 13,\ 2x + 2y + 10z = 14$ by Factorization method. (8)

2 a) Find the orthogonal projection of (1,1,1) on (2,4,4). Also represent the first vector as the sum of two orthogonal vectors. (7)

b) Let V be the vector space of polynomials with inner product $< f, g > = \int_0^1 f(t)g(t)dt$. Using Gram-Schmidt process find an orthonormal basis for V from the basis $\{1, t, t^2\}$. (8)

3 a) Derive Cauchy-Schwarz inequality. (7)

b) Find Singular Value Decomposition for the matrix $\begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}$ (8)

PART B

Answer any two full questions, each carries 15 marks

4 a) Evaluate $\Delta^4[(1-x)(1-2x)(1-3x)(1-4x)]$, where Δ is the forward difference operator. Assume the interval of difference being unity. (7)

b) Find the lowest degree polynomial which takes the following values (8)

x	0	1	2	3	4	5
$f(x)$	0	3	8	15	24	35

5 a) Find the minimum of the function $f(x) = (40 - 90x)^2$ using Golden section method in the interval [0,1] with n = 6 (Three iterations). (7)

b) Minimize $f(\lambda) = 0.65 - \frac{0.75}{1+\lambda^2} - 0.65\lambda\ tan^{-1}\frac{1}{\lambda}$ using Fibonacci method in the interval [0,3], using $n = 6$. (8)

6 a) Minimize $f(\lambda) = 2\lambda^2 + \frac{16}{\lambda}$ using Newton's direct root method starting point $\lambda_1 = 1, \epsilon = 0.04$. (Three iterations) (7)

b) From the following data, obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 500$ (8)

x	500	510	520	530	540	550
y	6.2146	6.2344	6.2538	6.2729	6.2916	6.3099

PART C

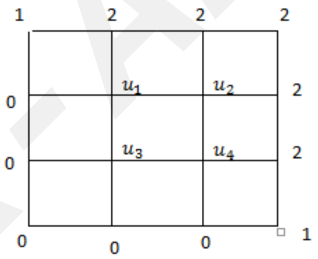
Answer any two full questions, each carries 20 marks

7 a) Minimise $f(x, y) = x^2 + 2y^2$ starting from the point $\begin{Bmatrix}1\\1\end{Bmatrix}$ using Fletcher-Reeve's method. (14)

b) Minimise $f(x_1, x_2) = 6{x_1}^2 + 2{x_2}^2 - 6x_1x_2 - x_1 - 2x_2$ starting from the point $X_1 = \begin{Bmatrix}0\\0\end{Bmatrix}$ using Newton's method. (6)

8 a) Solve the equation $u_t = u_{xx}$ subject to the conditions $u(x, 0) = \sin \pi x, 0 \le x \le 1, u(0, t) = u(1, t) = 0$. compute u for two time levels by taking $h = \frac{1}{3}, k = \frac{1}{36}$ using Crank-Nicolson Method. (10)

b) Solve $u_{xx} + u_{yy} = 0$ over the square mesh with boundary values as shown in the figure. Assuming $u_4 = 0.75$, iterate till the mesh values are correct up to two decimal places using Gauss- Seidel Method. (10)



9 a) Minimise $f(x_1, x_2) = x_1 - x_2 + 2{x_1}^2 + 2x_1x_2 + {x_2}^2$ starting from the point $X_1 = \begin{Bmatrix}0\\0\end{Bmatrix}$ using Univariate method. (Two iterations, take $\epsilon = 0.01$,). (10)

b) Evaluate the pivotal values of the equation $u_{tt} = 16u_{xx}$ taking $\Delta x = 1$ up to $t = 1.25$. The boundary conditions are $u(0, t) = u(5, t) = 0, \frac{\partial u}{\partial t}(x, 0) = 0$ and initial condition $u(x, 0) = x^2(5 - x)$. (10)