APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech examinations (S) September 2020 S1/S2 (2015 Scheme)

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 5 marks. Marks

1 a) Determine whether the series $\sum_{k=1}^{\infty}\left(\frac{2}{3}\right)^{k+2}$ converges. If so, find the sum (2)

b) Find the Maclaurin series expansion of $f(x) = \boldsymbol{ln}(\mathbf{1}-\boldsymbol{x})$up to 3 terms (3)

2 a) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $ye^x - 5cos2z = 3z$ (2)

b) Use chain rule to find $\frac{dw}{dx}$ at (0,1,2) for $w = xy + yz$, $y = \sin x, z = e^x$. (3)

3 a) Find the velocity of a particle moving along the curve $\vec{r}(t) == e^t \sin t\,\vec{i} + e^t \cos t\,\vec{j} + t\,\vec{k}$ at t = π (2)

b) Find the unit normal to the surface $yz + zx + xy = c$ at (-1,2,3) (3)

4 a) Evaluate $\int_1^2 \int_y^{3-y} dx\, dy$ (2)

b) Evaluate $\int_1^2 \int_0^x \frac{dy\,dx}{x^2+y^2}$. (3)

5 a) Find the value of constant a so that if $\bar{F} = (3x - 2y + z)\,\boldsymbol{i} + (4x - ay + z)\boldsymbol{j} + (x - y + 2z)\boldsymbol{k}$ is solenoidal. (2)

b) Find the work done by a force field $F(x, y) = -y\boldsymbol{i} + x\boldsymbol{j}$ acting on a particle moving along the circle $x^2 + y^2 = 3$ from $(\sqrt{3}, 0)$ to $(0, \sqrt{3})$ (3)

6 a) Determine the source and sink of the vector field $F(x, y, z) = 2(x^3 - 2x)\boldsymbol{i} + 2(y^3 - 2y)\boldsymbol{j} + 2(z^3 - 2z)\boldsymbol{k}$ (2)

b) Using Stoke's theorem prove that $\int_C \bar{r}\,.\,d\bar{r} = 0$ where $\bar{r} = x\,\boldsymbol{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$ and C is any closed curve. (3)

PART B

Module 1

Answer any two questions, each carries 5 marks.

7	Test the convergence of the infinite series $\sum_{k=1}^{\infty}\left(\frac{k}{k+1}\right)^{k^2}$.	(5)
8	Examine the convergence of $\sum_{k=0}^{\infty}\frac{(k+4)!}{4!k!4^k}$	(5)
9	Find the radius of convergence of the power series $\sum_{k=1}^{\infty}\frac{(2x-3)^k}{4^{2k}}$	(5)

Module 1I

Answer any two questions, each carries 5 marks.

10	The height and radius of a circular cone is measured with errors of atmost 3% and 5% respectively. Use differentials to approximate the maximum percentage error in calculated volume.	(5)
11	If $u = x^2 tan^{-1}\left(\frac{y}{x}\right) - y^2 tan^{-1}\left(\frac{x}{y}\right)$, find $\frac{\partial^2 u}{\partial x \partial y}$	(5)
12	Find relative extrema and saddle points, if any, of the function $f(x,y) = x^3 + y^3 - 15xy$.	(5)

Module 1II

Answer any two questions, each carries 5 marks.

13	Find where the tangent line to the curve $\boldsymbol{r}(t) = e^{-2t}i + \cos t\, j + 3\sin t\, k$ at the point (1,1,0) intersects the YZ plane.	(5)
14	Find the position and velocity vectors of the particle given **a**(t)=$(t+1)^{-2}$**j** -$e^{-2t}k$, **v**(0) = 3**i**-**j** , r(0)= k	(5)
15	A particle moves along a curve $x = 2t^2, y = t^2 - 4t, z = 3t - 5$ where t is the time. Find the component of acceleration at time t = 1 in the direction of $\vec{i} - 3\vec{j} + 2\vec{k}$	(5)

Module 1V

Answer any two questions, each carries 5 marks.

16	Evaluate $\iiint_R xysin\, z\, dV$ where R is the rectangular box defined by $0 \le x \le 1, 0 \le y \le 1, \quad 0 \le z \le \frac{\pi}{6}$	(5)
17	Sketch the region of integration and evaluate $\int_1^2 \int_y^{y^2} dx\, dy$ by changing the order of integration.	(5)
18	Use double integral to find the area bounded by the x – axis $y = 2x \;\; and \;\; x + y = 1$	(5)

Module V

Answer any three questions, each carries 5 marks.

19 Prove that $\int_C (x^2 - yz)\,\bar{\imath} + (y^2 - zx)\bar{\jmath} + (z^2 - xy)\bar{k}\ .\ d\bar{r}$ is independent of the path and evaluate the integral along any curve from (0,0,0) to (1,2,3). (5)

20 Evaluate $\int_C\ xy^2\,dx + xy\,dy$ where C is a triangle with vertices at $(0,0), (0,1)$ and (2,1) (5)

21 Evaluate $\int_C\ 2xy\,dx + (x^2 + y^2)dy$ along the curve $C: x = \cos t\,, y = \sin t$, $0 \leq t \leq \frac{\pi}{2}$ (5)

22 Determine whether $F(x, y) = 6y^2\,i + 12xy\,j$ is a conservative vector field. If so find the potential function for it. (5)

23 If$\bar{F} = (sinz + y\cos x)\boldsymbol{i}$ + $(\sin x + 2\cos y)\boldsymbol{j}$ $+(\sin y + x\cos z)\boldsymbol{k}$,find $Div\ \bar{F}$ and $Curl\ \bar{F}$. (5)

Module VI

Answer any three questions, each carries 5 marks.

24 Using Stoke's theorem, evaluate $\int_C\ \overline{F}\ .\ d\bar{r}$ where C is the boundary of the projection of the sphere$x^2 + y^2 + z^2 = 1$ on the XY plane with (5)

$$\bar{F} = (2x - y)\bar{\imath} - yz^2\bar{\jmath} - y^2z\,\bar{k}$$

25 Using Green's theorem evaluate $\int_C\ (y^2 - 7y)dx + (2xy + 2x)\,dy$ where C is the circle $x^2 + y^2 = 1$ (5)

26 Evaluate using divergence theorem for $\vec{F} = x^2i + zj + yzk$ taken over the cube bounded by $x = 0,\ x = 1, y = 0,\ y = 1,\ \ z = 0\ and\ z = 1$ (5)

27 Evaluate the surface integral $\iint_\sigma\ z^2\ ds$, where σ is the portion of the curve $z = \sqrt{x^2 + y^2}$ between $z = 1$ and $z = 3$ (5)

28 Use Green' theorem to find the area enclosed by the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ (5)

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