

Reg No.: \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**

B.Tech S1,S2 (S) Examination September 2020 (2015 Scheme)

**Course Code: MA102****Course Name: DIFFERENTIAL EQUATIONS**

Max. Marks: 100

Duration: 3 Hours

**PART A***Answer all questions, each carries 3 marks*

- 1 Find the ODE  $y'' + ay' + by = 0$  for the basis  $\{e^x, xe^x\}$  (3)
- 2 Reduce to first order and solve  $2xy'' = 3y'$ . (3)
- 3 Find the particular integral of  $y'' + 4y' + 4y = x^2$ . (3)
- 4 Using a suitable transformation, convert the differential equation  $(x^2D^2 - 4xD + 6)y = x$  into a linear differential equation with constant coefficients. (3)
- 5 If  $f(x)$  is a periodic function of period  $2L$  defined in  $[-L, L]$ . Write down Euler's Formulas  $a_0, a_n, b_n$  for  $f(x)$ . (3)
- 6 Find the Fourier series of the function  $f(x) = x$  in the range  $-\pi < x < \pi$ . (3)
- 7 Find the PDE by eliminating arbitrary constants  $a$  and  $b$  from  $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ . (3)
- 8 Find the particular integral of  $(D^3 - 4D^2D' + 4DD'^2)z = \cos(2x + y)$ . (3)
- 9 Write all possible solutions of one dimensional wave equation. (3)
- 10 A homogeneous string is stretched and its ends are fixed at  $x = 0$  and  $x = 40$ . Motion is started by displacing the string into the form  $f(x) = \sin\left(\frac{\pi x}{40}\right)$  from which it is released at time  $t = 0$ . Write the boundary and initial conditions.. (3)
- 11 Solve one dimensional heat equation for  $\lambda < 0$ . (3)
- 12 Find the steady state temperature distribution in a rod of length 40 cm if the ends are kept at  $0^\circ\text{C}$  and  $100^\circ\text{C}$ . (3)

**PART B***Answer six questions, one full question from each module***Module 1**

13 a) Solve  $y'' - 2y' - 3y = 0, y(-1) = e, y'(1) = -\frac{e}{4}$ . (6)

b) Show that the functions  $x^3$  and  $x^5$  are the basis of solutions of ODE  
 $x^2y'' - 7xy' + 15y = 0$ . (5)

**OR**

14 a) Solve ODE  $y^v - 3y^{iv} + 3y''' - y'' = 0$ . (6)

b) Solve the ODE  $xy'' + 2y' + xy = 0$ . Given that  $y_1 = \frac{\cos x}{x}$  is a solution. (5)

**Module 1I**

15 a) By the method of variation of parameters, solve  $y'' + y = \sec x$ . (6)

b) Solve  $x^2y'' - 4xy' + 6y = x^5$ . (5)

**OR**

16 a) Solve  $(2x + 3)^2y'' - 2(2x + 3)y' - 12y = 6x$ . (6)

b) Solve  $y'' + 2y' - 3y = e^{2x} \sin x$ . (5)

**Module 1II**

17 a) Find the Fourier series of  $f$  defined by  $f(x) = e^x$  in  $(-\pi, \pi)$ . (11)

**OR**

18 a) Obtain Fourier series for the function  $f(x) = x^2, -\pi \leq x \leq \pi$ . (6)

b) Expand  $f(x) = \cos x$  as a half range sine-series in  $0 \leq x \leq \pi$ . (5)

**Module 1V**

19 a) Solve  $r + s - 2t = \sqrt{2x + y}$ . (6)

b) Find the general solution of  $x^2p + y^2q = (x + y)z$ . (5)

**OR**

20 a) Solve  $4r + 12s + 9t = e^{3x-2y}$ . (6)

b) Solve  $(D^2 - DD' - 6D'^2)z = xy$ . (5)

**Module V**

21 a) Using method of separation of variables, solve  $y^2u_x - x^2u_y = 0$ . (5)

b) Find the displacement of a finite string of length  $l$  that is fixed at both ends and is released from rest with an initial displacement of  $2 \sin\left(\frac{\pi x}{l}\right)$ . (5)

OR

- 22 Derive one dimensional wave equation. (10)

## Module VI

- 23 A rod of length  $L$  is heated so that its ends A and B are at zero temperature .If its initial temperature is given by  $u = \frac{cx(L-x)}{L^2}$ , find the temperature at time  $t$ . (10)

OR

- 24 A rod of length 40cm has its ends A and B kept at  $0^{\circ}\text{C}$  and  $100^{\circ}\text{C}$  respectively until steady state conditions prevail. Suddenly the temperature at A is raised to  $20^{\circ}\text{C}$  and the end B is decreased to  $60^{\circ}\text{C}$  . Find the temperature distribution in the rod at time  $t$ . (10)

\*\*\*\*