

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
V SEMESTER B.TECH DEGREE EXAMINATION(S), MAY 2019

Course Code: CS309

Course Name: GRAPH THEORY AND COMBINATORICS

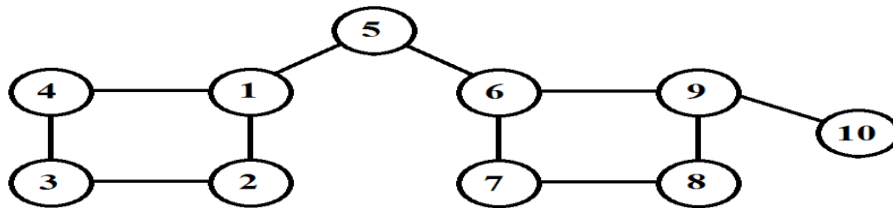
Max. Marks: 100

Duration: 3 Hours

PART A

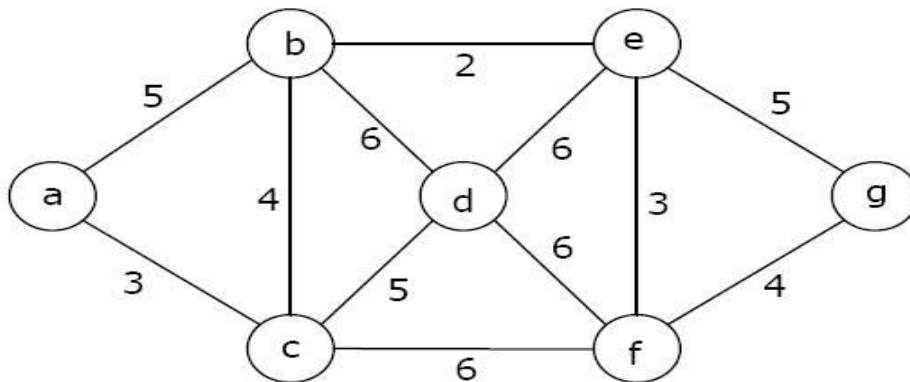
Answer all questions, each carries 3 marks.

- | | | |
|---|---|---|
| 1 | Print a Walk, trail, path and cycle on the graph below. | 3 |
|---|---|---|



- | | | |
|---|---|---|
| 2 | Define pendant vertex, isolated vertex and null graph with an example each. | 3 |
|---|---|---|

- | | | |
|---|---|---|
| 3 | State travelling salesman problem. Print a travelling salesman's tour on the graph below. | 3 |
|---|---|---|

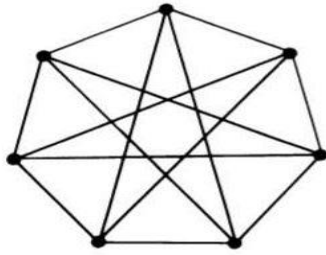


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|---|--|---|
| 4 | Prove Dirac's theorem for Hamiltonicity. | 3 |
|---|--|---|

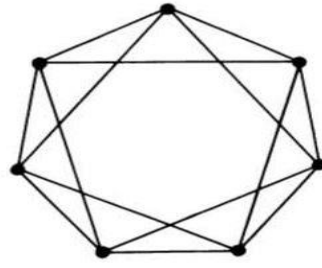
PART B

Answer any two full questions, each carries 9 marks.

- | | | |
|---|---|---|
| 5 | a) Define isomorphism of graphs. Show that the graphs (a) and (b) are isomorphic. | 4 |
|---|---|---|

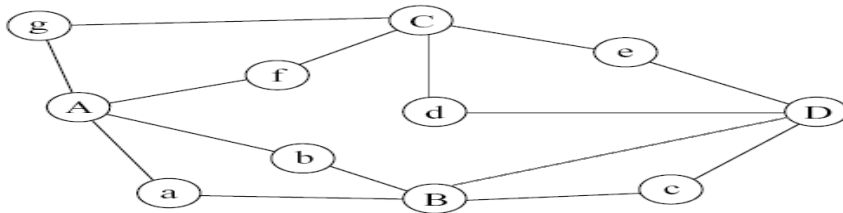


(a)

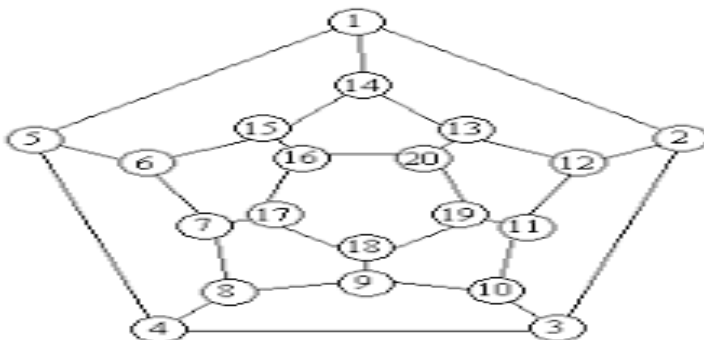


(b)

- b) Define subgraph. Give two subgraphs of the above graph.(Fig. a) 2
- c) Consider a complete graph G with 11 vertices. 3
 - 1. Find the maximum number of edges possible in G
 - 2. Find the number of edge-disjoint Hamiltonian circuits in G
- 6 a) Draw a simple disconnected graph with 10 vertices, 4 components and maximum 3
number of edges.
- b) Explain any two applications of graphs. 2
- c) Check whether the given graph is an Euler graph and if yes, give the Euler line. 4
Justify your answer.



- 7 a) Prove or disprove: If every vertex of a simple graph G has degree 2, then G is a 3
cycle.
- b) Give Hamiltonian circuit of the following graph. 3

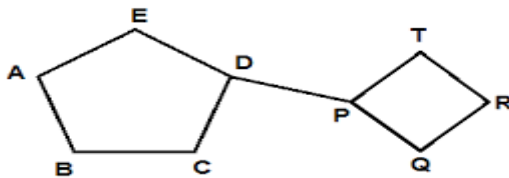


- c) In a graph G let p_1 and p_2 be two different paths between two given vertices. 3
Prove that ringsum of p_1 and p_2 is a circuit or a set of circuits.

PART C

Answer all questions, each carries 3 marks.

- 8 Prove that in a graph G , if there is exactly one path between every pair of vertices, then G is a tree. 3
- 9 Given a spanning tree of a graph, how will you find out all spanning trees? 3
- 10 List all cut sets of the following graph. 3



- 11 Prove that every circuit has an even number of edges in common with any cut set. 3

PART D

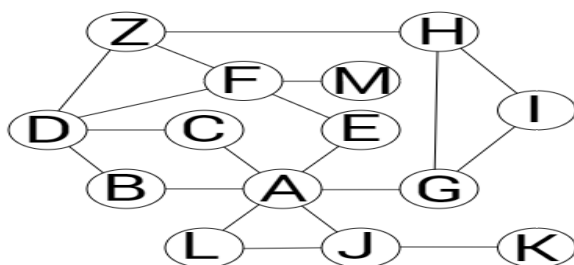
Answer any two full questions, each carries 9 marks.

- 12 a) Define a tree. Give any 4 properties of trees. 3
- b) Prove that a graph is a tree if and only if it is loop-less and has exactly one spanning tree. 3
- c) Prove that every circuit has an even number of edges in common with any cut set. 3
- 13 a) Prove that every tree has either one or two centers. 3
- b) Write short notes on geometric dual and combinatorial dual. 6
- 14 a) Draw a connected graph G and find two spanning trees T_1 and T_2 of G such that the distance $(T_1, T_2) = 3$. Find the branch set, chord set, rank and nullity of T_1 . 4
- b) Construct a graph G with the following properties: Edge connectivity = 4, Vertex connectivity = 3 and degree of every vertex of G is greater than or equal to 5. 5

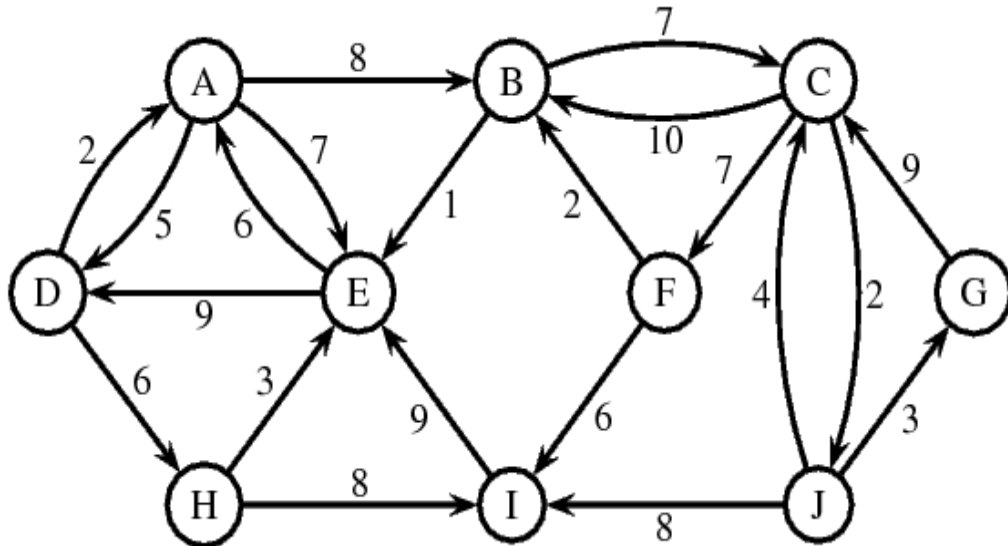
PART E

Answer any four full questions, each carries 10 marks.

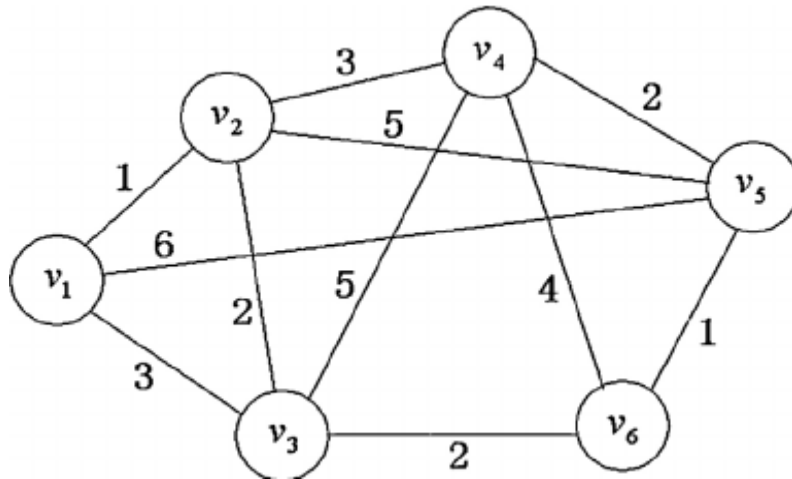
- 15 a) Give incidence matrix of the following graph. 3



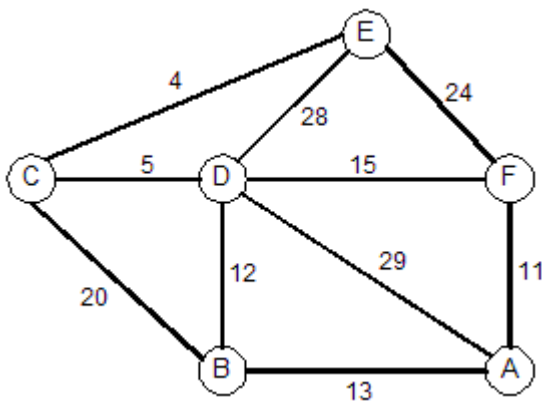
- b) Prove that two graphs G_1 and G_2 are isomorphic if and only if their incidence matrices $A(G_1)$ and $A(G_2)$ differ only by permutations of rows and columns. 2
- c) Give Dijkstra's algorithm to find shortest path between a vertex pair. Use it to find shortest path between A and G. 5



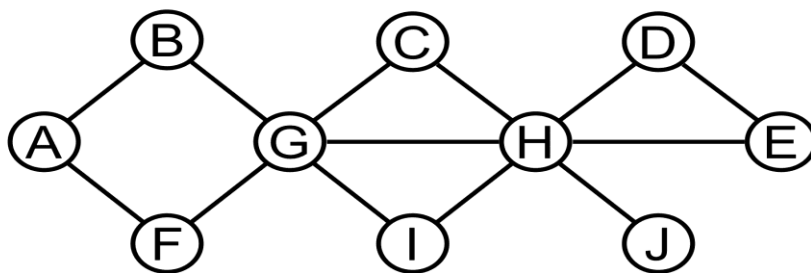
- 16 a) Prove that if B is a circuit matrix of a connected graph G with n vertices and e edges, then rank of B is $e-n+1$. 3
- b) How will you get fundamental circuit matrix from a circuit matrix. Derive the rank of a fundamental circuit matrix. 2
- c) Explain successor listing and incidence matrix methods used in computer representation of a graph? 5
- 17 a) Prove that the rank of cut set matrix $C(G)$ is equal to rank of the incidence matrix $A(G)$, which equals the rank of the graph G . 3
- b) Define path matrix. What is the disadvantage of path matrix compared to other matrices. 2
- c) Find a minimum spanning tree of the following graph. Also give its rank and nullity. 5



- 18 a) If $A(G)$ is an incidence matrix of a connected graph G with n vertices, then the rank of $A(G)$ is $n-1$. 5
- b) How is Kruskal's algorithm used to find minimum cost spanning tree of a graph. Find a minimum spanning tree in the graph below. 5



- 19 a) Write cut set matrix of the following graph. Give its rank. 5



- b) Give an algorithm to check whether a graph is connected or not. How it can be implemented with an adjacency matrix. 5
- 20 a) Give any five properties of circuit matrix. 5
- b) How are edge listing and linear arrays used in computer representation of a graph? 5