Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER B.TECH DEGREE EXAMINATION (S), MAY 2019

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 5 marks. Marks

1 a) Check the convergence of the series $\sum_{k=1}^{\infty}\left(\frac{3k-4}{4k-5}\right)^k$ (2)

b) Find the Maclaurin series of $f(x) = \frac{1}{1+x}$, up to 3 terms (3)

2 a) If $z = (3x-2y)^4$, find $\frac{\partial^4 z}{\partial x\, \partial y^3}$ (2)

b) If $w = \log(\tan x + \tan y + \tan z)$ then prove that (3)
$\sin 2x \frac{\partial w}{\partial x} + \sin 2y \frac{\partial w}{\partial y} + \sin 2z \frac{\partial w}{\partial y} = 2$

3 a) Find the speed of a particle moving along the path $x = 2\cos t, y = 2\sin t, z = t$ (2)
at t = π/2

b) If $y'(t) = \cos t\ i + \sin t\ j\ ;\ \ y(0) = i - j$. Find $y(t)$. (3)

4 a) Evaluate $\int_0^1 \int_0^{x^2} \int_0^2 dy\, dz\, dx$ (2)

b) Evaluate $\iint xy$ dx dy over the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and lying in the first quadrant. (3)

5 a) Show that $F(x, y) = 2xy^3 i + 3x^2y^2 j$ is conservative. (2)

b) If $\bar{r} = x\,\bar{i} + y\,\bar{j} + z\,\bar{k}$ and $r = \|\bar{r}\|$, prove that $\nabla . \frac{\bar{r}}{r^3} = 0$ (3)

6 a) Evaluate by Stoke's theorem $\oint_C (e^x\, dx + 2y\, dy - dz)$, where C is the curve (2)
$x^2 + y^2 = 4,\ z = 2$

b) Using Green's theorem evaluate $\int_C x\, dy - y\, dx$ where C is the circle $x^2 + y^2 = 4$ (3)

PART B

Module 1

Answer any two questions, each carries 5 marks.

7 Test for convergence of the series $\sum_{k=1}^{\infty} \frac{1}{(8k^2-3k)^{1/3}}$ (5)

8 Find the radius of convergence and interval of convergence of the power series $\sum_{k=0}^{\infty} \frac{(2x-1)^k}{3^{2k}}$. (5)

9 Show that the series $\sum_{k=1}^{\infty}(-1)^k\left(\frac{k}{k+1}\right)^{k^2}$ is convergent. (5)

Module 1I

Answer any two questions, each carries 5 marks.

10 Let $w = \sqrt{x^2+y^2+z^2}$, where $x - \cos\theta, y - \sin\theta,\ z - \tan\theta$. Find $\frac{dw}{d\theta}$ at $\theta = \frac{\pi}{4}$, using chain rule. (5)

11 Find the local linear approximation $L(x,y)$ to $f(x,y) = \ln\ (xy\ )$ at the point $P(1{,}2)$. Compare the error in approximating f by L at the point $Q(1.01{,}2.01)$ with the distance between P and Q . (5)

12 Find relative extrema and saddle points, if any, of the function $f(x,y) = xy + \frac{8}{x} + \frac{8}{y}$. (5)

Module 1II

Answer any two questions, each carries 5 marks.

13 Find the unit tangent T(t) and unit normal N(t) to the curve $x = a\cos t,\ y = a\sin t,\ z = ct$ a >0 (5)

14 Find the velocity and position vectors of the particle ,if the acceleration vector $a(t) = \sin t\, i + \cos t\, j + e^t\, k$; $v(0) = k$; $r(0) = -i + k$. (5)

15 Find the equation of the tangent line to the curve of intersection of surfaces $z = x^2 + y^2$ and $3x^2 + 2y^2 + z^2 = 9$ and the point (1,1,2). (5)

Module 1V

Answer any two questions, each carries 5 marks.

16 Evaluate by reversing the order of integration $\int_0^{a/\sqrt{2}} \int_y^{\sqrt{a^2-y^2}} x\,dx\,dy$ (5)

17 Evaluate $\iint_R xy\,dA$, where R is the sector in the first quadrant bounded by $y = \sqrt{x},\ y = 6 - x,\ y = 0$. (5)

18 Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x}$ x dz dx dy (5)

Module V

Answer any three questions, each carries 5 marks.

19 Find the work done by $F(x, y) = (x^2 + y^2)i - xj$ along the curve $C: x^2 + y^2 = 1$ counter clockwise from $(1,0)$ to $(0,1)$ (5)

20 Determine whether $F(x, y) = 6y^2\, i + 12xy\, j$ is a conservative vector field. If so find the potential function for it. (5)

21 Find the divergence and curl of the vector field $F(x, y, z) = xyz^2 i + yzx^2\, j + zxy^2\, k$ (5)

22 Prove that $\int_C (x^2 - yz)\bar{\imath} + (y^2 - zx)\bar{\jmath} + (z^2 - xy)\bar{k}\,.\,d\bar{r}$ is independent of the path and evaluate the integral along any curve from (0,0,0) to (1,2,3). (5)

23 If $\bar{r} = x\,\bar{\imath} + y\,\bar{\jmath} + z\,\bar{k}$ and $r = \|\bar{r}\|$, prove that $\nabla^2 f(r) = \frac{2}{r} f'(r) + f''(r)$. (5)

Module VI

Answer any three questions, each carries 5 marks.

24 Using Green's theorem evaluate $\int_c (xy + y^2)dx + x^2\,dy$ where C is the boundary of the region bounded by $y = x^2$ and $x = y^2$ (5)

25 Evaluate the surface integral $\iint_\sigma z^2\ ds$, where σ is the portion of the curve $z = \sqrt{x^2 + y^2}$ between $z = 1$ and $z = 3$ (5)

26 Determine whether the vector field $F(x, y, z)$ is free of sources and sinks. If not, locate them. (5)

$i) F(x, y, z) = (y + z)\bar{\imath} - xz\bar{\jmath} + x^2 \sin y\ \bar{k}$

$ii) F(x, y, z) = x^3\bar{\imath} + y^3\bar{\jmath} + 2z^3\bar{k}$

27 Use divergence theorem to find the outward flux of the vector field $F(x, y, z) = (2x + y^2)i + xy\, j + (xy - 2z)k$ across the surface σ of the tetrahedron bounded by $x + y + z = 2$ and the coordinate planes. (5)

28 Using Stoke's theorem evaluate $\int_C \bar{F}\,.\,d\bar{r}$; where $\bar{F} = xy\,\bar{\imath} + yz\,\bar{\jmath} + xz\,\bar{k}$; C triangular path in the plane $x + y + z = 1$ with vertices at $(1,0,0)$, $(0,1,0)$ and (0,0,1) in the first octant (5)

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