Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

SECOND SEMESTER B.TECH DEGREE EXAMINATION (R & S), MAY 2019

Course Code: MA102

Course Name: DIFFERENTIAL EQUATIONS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks

1	Find the general solution of $\frac{d^3y}{dx^3} + y = 0$	(3)
2	Find the Wronskian of $e^x \cos 2x$ and $e^x \sin 2x$	(3)
3	Find the Particular Integral of . $y'' - 4y' - 5y = 4\,cos2x.$	(3)
4	Find the particular integral of $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = \sinh 2x$	(3)
5	Evaluate the coefficient a_n in the Fourier series expansion for $f(x) = \lvert\sin x\rvert$ in $-\pi < x < \pi$	(3)
6	Find the half range Fourier sine series representation of $f(x) = k$ in $(0,\pi)$	(3)
7	Find the partial differential equation of all spheres having their centre lies on z-axis.	(3)
8	Form the partial differential equation of $z = f(\frac{xy}{z})$ by eliminating the arbitrary function f.	(3)
9	Solve $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}, u(0,y) = 8e^{-3y}$, using the method of separation of variables.	(3)
10	A tightly stretched string of length l is fixed at both ends and pulled from its mid point to a height h and realised from rest from this position. Write down the initial and boundary conditions.	(3)
11	Find the steady state temperature distribution in a rod of length 30 cm, if the ends of the rod are kept at $20^\circ C$ and $80^\circ C$	(3)
12	Write down the three possible solutions of the one dimensional heat equation.	(3)

PART B

Answer six questions,one full question from each module

Module 1

13 a) Solve the initial value problem $y'' + 4y' + 5y = 0, y(0) = 2, y'(0) = -5.$ (6)

b) Find the general solution of the differential equation $y''' - y'' + 4y' = 0$ (5)

OR

14 a) If $y_1(x) = x$ is a solution to the differential equation $(1 + x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$, find the general solution. (6)

b) Solve the ordinary differential equation $y''' - 3y'' - 4y' + 6y = 0.$ (5)

Module 1I

15 a) Solve $2(3x + 1)^2\frac{d^2y}{dx^2} + 21(3x + 1)\frac{dy}{dx} + 18y = 9x$ (6)

b) Solve $(D^4 + 2D^2 + 1)y = x^4$ (5)

OR

16 a) Use method variation of parameters to solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$ (6)

b) Solve $(D^2 - 4D + 4)y = sin^2$ x (5)

Module 1II

17 a) Obtain the half range Fourier cosine series expansion of $f(x) = x \sin x$ in (0,π) . (6)

b) Find the Fourier series for $f(x) = |x|$, -π<x<π (5)

OR

18 a) Find the Fourier series for $f(x) = \begin{cases} 0, -\pi < x < 0 \\ \pi, 0 < x < \pi \end{cases}$ (6)

b) Find the Fourier series of the periodic function f(x) of period 4, where $f(x) = \begin{cases} 0, -2 < x \leq -1 \\ k, -1 < x < 1 \\ 0, 1 \leq x < 2 \end{cases}$ (5)

Module 1V

19 a) Solve $\frac{y^2z}{x}p + xzq = y^2$ (6)

b) Find the partial differential equation of all planes which are at a constant distance k from the origin. (5)

OR

20 a) Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$ (6)

b) Solve $(D^2 + 3DD' + 2D'^2)z = x^2y^2$ (5)

Module V

21 A string is stretched between two fixed points at a distance of 60 cm and the points of the string are given initial velocities where

$$v = \begin{cases} \frac{\lambda x}{30}, & 0 < x < 30 \\ \frac{\lambda}{30}(60 - x), & 30 < x < 60 \end{cases}$$

, x being the distance from an end, find the displacement at any time t. (10)

OR

22 A uniform elastic string of length 60 cm is subjected to a constant tension of 2 Kg. If the ends are fixed, the initial displacement $u(x, 0) = 60x - x^2, 0 < x < 60$ and the initial velocity is zero, find the displacement function $u(x, t)$ (10)

Module VI

23 Find the temperature distribution in a rod of length 2m whose end points are maintained at temperature $0^\circ C$ and the initial temperature is $f(x) = 100(2x - x^2), 0 \leq x \leq 2$ (10)

OR

24 A bar 10 cm long with insulated sides has its ends A and B maintained at $50^\circ C$ and $100^\circ C$ respectively until steady state conditions prevail. The temperature of A is suddenly raised to $90^\circ C$ and at the same time that at B is lowered to $60^\circ C$. Find the temperature distribution in the bar at time t. (10)

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