Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S1,S2 (S,FE) Examination May 2021 (2015 Scheme)

Course Code: MA102

Course Name: DIFFERENTIAL EQUATIONS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all Questions. Each question carries 3 Marks Marks

1	Solve the ODE, $y''' + y = 0$	(3)
2	Show that e^{3x} and e^{2x} are linearly independent solutions of $y" - 5y' + 6y = 0$	(3)
3	Solve,$(D^2 + 3D + 2)y = 5$	(3)
4	Using a suitable transformation, convert the differential equation $(1 + x)^2y" + (1 + x)y' = (2x + 3)(2x + 4)$ into a linear differential equation with constant coefficients	(3)
5	Represent the function $f(x) = x^2$ as a Fourier series in the interval$(-\pi, \pi)$	(3)
6	Find the half range Fourier sine series of $f(x) = e^x$ in $0 < x < 1$	(3)
7	Form a PDE by eliminating the arbitrary function from $z = y^2 + 2f(\frac{1}{x} + logy)$	(3)
8	Find the P.I. of $\left(D^2 - 5DD' + 4D'^2\right)z = \sin(4x + y)$	(3)
9	Solve $x\frac{\partial u}{\partial x} - 2y\frac{\partial u}{\partial y} = 0$ using method of separation of variables.	(3)
10	A tightly stretched flexible string has its ends at $x = 0$ and $x = l$. At time $t = 0$, the string is given a shape defined by $f(x) = \mu x(l - x)$, where μ is a constant, and then released. Write the boundary and initial conditions	(3)
11	Write down the possible solutions of one dimensional heat equation.	(3)
12	The ends A and B of a rod of length l have the temperature a^0C and b^0C respectively until steady state conditions prevail. Find the initial temperature distribution of the rod.	(3)

PART B

Answer six questions, one full question from each module

Module I

13 a) Find a basis of solutions of the ODE, $x^2y'' + xy' - 4y = 0$. Given $y_1 = x^2$ is one solution. (6)

b) Determine all possible solutions to the initial value problem, $y' = 1 + y^2$, $y(0) = 0$ in the interval $|x| < 3, and\ |y| < 2$ (5)

OR

14 a) Verify by substitution that $y_1 = e^{-x}cosx\ and\ y_2 = e^{-x}sinx$ are the solutions of the given ODE and then solve the initial value problem, $y'' + 2y' + 2y = 0$, $y(0) = 0, y'(0) = 15$ (6)

b) Find the general solution of $y^{iv} - 2y''' + 2y'' - 2y' + y = 0$ (5)

Module II

15 a) Solve, by the method of variation of parameters , $(D^2 + 1)y = cosecx$ (6)

b) Solve,$(D^3 - D^2 - 6D)y = x^2 + 1$ (5)

OR

16 a) Solve,$(D^2 - 2D + 5)y = e^{2x}sinx$ (6)

b) Solve, $x^2\frac{d^3y}{dx^3} - 4x\frac{d^2y}{dx^2} + 6\frac{dy}{dx} = 4$ (5)

Module III

17 Find the Fourier series of $f(x) = \begin{Bmatrix} 0, & -\pi < x < 0 \\ sinx, & 0 < x < \pi \end{Bmatrix}$ (11)

OR

18 a) Find the Fourier series of $f(x) = \begin{Bmatrix} \pi x, & 0 < x < 1 \\ \pi(2 - x), & 1 < x < 2 \end{Bmatrix}$ (6)

b) Obtain the half range Fourier cosine series of $f(x) = (x - 1)^2, 0 < x < 1$. (5)

Show that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ \dots\dots\dots\dots\dots\dots.. = \frac{\pi^2}{6}$

Module IV

19 a) Solve, $r - 4s + 4t = e^{2x+y}$ (6)

b) Solve, $(2z - y)p + (x + z)q = -2x - y$ (5)

OR

20 a) Solve, $(D^2 - 2DD' - 15D'^2)z = 12xy$ (6)

b) Find the PDE of all planes cutting equal intercepts from the X and Y axes. (5)

Module V

21 a) Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x,0) = 6e^{-3x}$. (5)

b) Find the displacement of a finite string of length L that is fixed at both ends and is released from rest with an initial displacement $f(x)$. (5)

OR

22 Derive one dimensional wave equation. (10)

Module VI

23 A rod of length l is heated so that its ends A and B are at zero temperature. If initially its temperature is given by $u = \frac{cx(l-x)}{l^2}$, find the temperature distribution at time t. (10)

OR

24 A long iron rod, with insulated lateral surface has its left end maintained at a temperature of $0^\circ C$ and its right end at $x = 2$ maintained at $100^\circ C$. Determine the temperature as a function of x and t if the initial temperature is (10)

$$u(x,t) = \begin{cases} 100x, & 0 < x < 1 \\ 100, & 1 < x < 2 \end{cases}$$

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