Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

First Semester B.Tech Degree Regular and Supplementary Examination December 2020 (2019 Scheme)

Course Code: MAT101
Course Name: LINEAR ALGEBRA AND CALCULUS
(2019 Scheme)

Max. Marks: 100 Duration: 3 Hours

PART A
Answer all questions, each carries 3 marks.

1	Determine the rank of the matrix $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 2 & 4 & -3 \end{bmatrix}$	(3)
2	Show that the quadratic form $4x^2 + 12xy + 13y^2$ is positive definite.	(3)
3	If $z = \sin(y^2 - 4x)$ find the rate of change of z with respect to x at the point $(3,1)$ with y held fixed.	(3)
4	Find $\frac{dz}{dt}$ by chain rule ,where $z = 3x^2 y^2$, $x = t^4, y = t^3$	(3)
5	Find the mass of the lamina with density function x^2 which is bounded by $y = x$ and $y = x^2$.	(3)
6	Evaluate $\iint_R y^2 x\, dA$ over the region $R = \{(x,y), -3 \le x \le 2, 0 \le y \le 1\}$	(3)
7	Test the convergence of the series $\sum_{k=1}^{\infty} \left(\frac{k}{100}\right)^k$	(3)
8	Does the series $\sum_{k=1}^{\infty} \left(\frac{-3}{4}\right)^k$ converge? If so, find the sum.	(3)
9	Find the binomial series for $f(x) = (1+x)^{1/3}$ up to third degree term.	(3)
10	Find the Maclaurin's series of $f(x) = \log(1+x)$ up to third degree term.	(3)

PART B
Answer one full question from each module, each question carries 14 marks

Module-I

11	a)	Solve the following linear system of equations using Gauss elimination method. $x + y + z = 6$, $x + 2y - 3z = -4$, $-x - 4y + 9z = 18$	(7)

b) Find eigen values and eigen vectors of the matrix (7)

$$A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$

12 a) Show that the equations:

$x + y + z = a, \quad 3x + 4y + 5z = b, \quad 2x + 3y + 4z = c$

(i)have no solution if $a = b = c = 1$ (7)

(ii)have many solutions if $a = \frac{b}{2} = c = 1$

b) Find the matrix of transformation that diagonalize the matrix (7)

$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. Also, find the diagonal matrix.

Module-II

13 a) Find the local linear approximation of $\frac{4y}{x+z}$ at $(1,1,1)$ (7)

b) Find the absolute extrema of the function $f(x, y) = x^2 - 3y^2 - 2x + 6y$ over the square region with vertices $(0,0), (0,2)$ $(2,2)$ and $(2,0)$. (7)

14 a) If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$. (7)

b) Locate all relative extrema of $f(x, y) = 2xy - x^3 - y^2$ (7)

Module-III

15 a) Use double integrals to find the area of the region enclosed between the parabola $2y = x^2$ and the line $y = 2x$ (7)

b) Find the volume of the solid in the first octant bounded by the coordinate planes and the plane $x + 2y + z = 6$. (7)

16 a) Change the order of integration and hence evaluate $\int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} dy\,dx$ (7)

b) Evaluate $\iiint_G z\,dV$, where G is the wedge in the first octant cut off from the cylindrical solid $y^2 + z^2 \leq 1$ and the planes $y = x$ and $x = 0$. (7)

Module-IV

17 a) Test the convergence of the series (7)

$$1+\frac{1.3}{3!}+\frac{1.3.5}{5!}+\frac{1.3.5.7}{7!}+\ldots\ldots\ldots$$

b) Find the sum of the series $\sum_{k=1}^{\infty}\frac{1}{k(k+1)}$ (7)

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a) Test the convergence of $(i)\ \sum_{k=1}^{\infty}\frac{k!}{3!\,(k-1)!\,3^k}$ $\quad (ii)\ \sum_{k=1}^{\infty}\left(\frac{4k-5}{2k+1}\right)^k$ (7)

b) Test the absolute or conditional convergence of $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^2}{k^3+1}$ (7)

Module-V

19 a) Expand into a Fourier series, $f(x) = e^{-x}\,,\ 0 < x < 2\pi$ (7)

b) Find the half range cosine series for $f(x)=(x-1)^2$ in $0 \le x \le 1$. (7)

20 a) Find the Fourier series of the function $f(x) = |x|$ in $-1 \le x \le 1$ (7)

b) Find the Fourier sine series of $f(x) = x\cos x$ in $0 < x < \pi$ (7)

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