Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S1,S2(S,FE) Examination May 2021 (2015 Scheme)

Course Code: MA101
Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A
Answer all Questions. Each question carries 5 Marks

			Marks
1	a)	Determine whether the series $\sum_{k=0}^{\infty}\left(\frac{4}{5}\right)^{k+2}$ converges.	(2)
	b)	Find the Maclaurins series of $f(x) = \frac{1}{1-x}$.	(3)
2	a)	Prove that $\frac{\partial^2 z}{\partial x\partial y} = \frac{\partial^2 z}{\partial y\partial x}$ where $z = x^2y$.	(2)
	b)	Compute the differential dz of the function $z = \frac{x+y}{xy}$.	(3)
3	a)	Find the velocity and acceleration of a particle moving along a circular path $\vec{r}(t) = 2cost\hat{\imath} + 2sint\hat{\jmath}$ at time $t = \frac{\pi}{4}$.	(2)
	b)	Fid the directional derivate of $f(x,y,z) = x^2y - yz^2 + z$ at the point $(1,-2,0)$ in the direction of the vector $\vec{a} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$.	(3)
4	a)	Evaluate $\int_0^1\int_0^1\int_0^1 e^{x+y+z}dxdydz$	(2)
	b)	Find the area of the region enclosed between the parabola $y = x^2$ and the line $y = 2x$.	(3)
5	a)	Find the divergence and curl of $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$.	(2)
	b)	Find the work done by the force field $\vec{F}(x,y,z) = xy\hat{\imath} + yz\hat{\jmath} + xz\hat{k}$ on a particle that moves along the curve $C: \vec{r}(t) = t\hat{\imath} + t^2\hat{\jmath} + t^3\hat{k}, (0 \le t \le 1)$.	(3)
6	a)	Evaluate by Green's Theorem, $\oint_C ydx + xdy$ where C is the unit circle.	(2)
	b)	Use Divergence theorem for $\vec{F}(x,y,z) = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ taken over the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$.	(3)

PART B
Module I
Answer any two questions. Each question carries 5 Marks

7	Test the convergence of $\frac{1}{1.2.3} + \frac{1}{3.4.5} + \frac{1}{5.6.7} + \cdots$	(5)
8	Find the Taylor series expansion of $f(x) = cosx; x = \frac{\pi}{2}$	(5)

9 Find the radius of convergence and interval of convergence of the series $\sum_{k=1}^{\infty} \frac{(x-5)^k}{5^k}$ (5)

Module II

Answer any two questions. Each question carries 5 Marks

10 Find the local linear approximation of $f(x,y) = \sqrt{x^2 + y^2}$ at (3,4) and compare the error in approximation by $L(3.04, 3.98)$ with the distance between the points. (5)

11 If $u = f(x-y, y-z, z-x)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. (5)

12 Find the relative extrema of $f(x,y) = xy - x^3 - y^2$ (5)

Module III

Answer any two questions. Each question carries 5 Marks

13 The position function of a particle is given by $\vec{r} = (t^3 - 2t)\hat{\imath} + (t^2 - 4)\hat{\jmath}$. Find the scalar tangential and normal components of acceleration. Also find the vector tangential and normal components of acceleration at $t = 1$. (5)

14 Prove that $\nabla f(r) = \frac{f'(r)}{r}\vec{r}$ where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = |\vec{r}|$. (5)

15 Find an equation of the tangent plane to the ellipsoid $x^2 + 4y^2 + z^2 = 18$ at the point (1,2,1) and the parametric equation for the normal line to the surface at the point. (5)

Module IV

Answer any two questions. Each question carries 5 Marks

16 Evaluate $\iint_R y\,dx\,dy$ where R is the region bounded by the parabola $y^2 = 4x$ and $x^2 = 4y$. (5)

17 Change the order of integration and evaluate $\int_0^2 \int_{y/2}^1 e^{x^2} dx\,dy$. (5)

18 Find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $x + z = 5$. (5)

Module V

Answer any three questions. Each question carries 5 Marks

19 Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$ where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$. (5)

20 Evaluate the line integral $\int_C -y\,dx + x\,dy$ along $y^2 = 3x$ from (3,3) to (0,0) (5)

21 Find the work done by the force field $\vec{F} = (y - x^2)\hat{\imath} + (z - y^2)\hat{\jmath} + (x - z^2)\hat{k}$ along the curve $\vec{r} = t\hat{\imath} + t^2\hat{\jmath} + t^3\hat{k}, 0 \le t \le 1$ from (0,0,0) to (1,1,1) (5)

22 Show that $\vec{F} = yz\hat{\imath} + xz\hat{\jmath} + yx\hat{k}$ is conservative field and hence find its scalar potential function. (5)

23 Show that the integral $\int_C (xy^2 + y^3)dx + (x^2y + 3xy^2)dy$ is independent of path joining the points (1,2) and (3,4). Hence evaluate the integral. (5)

Module VI

Answer any three questions. Each question carries 5 Marks

24 Use Green's theorem to evaluate the integral $\int_C (e^x + y^2)dx + (e^y + x^2)dy$ where C is the boundary of the region between $y = x^2$ and $y = 2x$. (5)

25 Evaluate the surface integral $\iint_\sigma xzds$ where σ is the part of the plane $x + y + z = 1$ that lies in the first octant. (5)

26 Use Stoke's theorem to evaluate $\int_C \vec{F}.d\vec{r}$ where $\vec{F}(x,y,z) = (x-y)\hat{\imath} + (y-z)\hat{\jmath} - (z-x)\hat{k}$ where C is the portion of the plane $x + y + z = 1$ in the first octant. Assume that the surface has an upward orientation (5)

27 Use the divergence theorem to find the outward flux of the vector field $\vec{F}(x,y,z) = (x-z)\hat{\imath} + (y-x)\hat{\jmath} + (2z-y)\hat{k}$; where S is the surface of the cylindrical solid bounded by $x^2 + y^2 = a^2, z = 0$ and $z = 1$. (5)

28 Evaluate $\iint \vec{F}.\hat{n}ds$ using divergence theorem, where $\vec{F} = 4xz\hat{\imath} + xyz^2\hat{\jmath} + 3z\hat{k}$ over the region bounded by the cone $z^2 = x^2 + y^2$ and the plane $z = 4$, above the xy-plane. (5)

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