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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH DEGREE EXAMINATION, APR 2021/DEC 2021

Branch: Mechanical Engineering

Streams: (i) Industrial Engineering (ii) Financial Engineering

01MA6017 Probability and Stochastic Processes

Max. marks: 60 Duration: 3 hours

PART A

(Answer any **two** questions. Each question carries **9 marks**.)

1. Suppose random variables X and Y have joint probability density given by:

$$f(x,y) = \begin{cases} 2, & 0 \leq x, y \leq 1, \quad x + y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the marginal densities (4)

   (b) Find $P(X > \frac{1}{2} | Y = \frac{1}{4})$ and $E[X|Y]$ (5)

2. Let X and Y be independent random variables with

$$P(X = 0) = P(X = 1) = \frac{1}{2}; \quad P(Y = 0) = P(Y = 2) = \frac{1}{2}$$

   (a) Compute the joint distribution X and Y. (3)

   (b) Compute the probability distribution of $Z = X + Y$ (6)

3. (a) X and Y are discrete random variables with joint probability mass function (5)

	$Y = 1$	$Y = 2$	$Y = 3$
$X = 1$	0.25	0.25	0
$X = 2$	0	0.25	0.25

   Find the correlation coefficient of X and Y and interpret the result.

   (b) Suppose that the service time (in minutes) for a customer at a bank counter is a random variable with mean 2 and variance 1. Assume that service times for different customers are independent. Using central limit theorem, find the probability that the total service time for 50 customers is between 90 and 110 minutes. (4)

PART-B

(Answer any **two** questions. Each question carries **9 marks**.)

4. A rapid transit system has just started operating. In the first month of operation, it was found that 25 % of commuters are using the system while 75 % are travelling by automobile. Suppose that each month 10 % of transit users go back to using their cars, while 30 % of automobile users switch to the transit system.

(a) Describe the above system using an appropriate Markov model and write the transition probability matrix. (3)

(b) What will be the proportion of people using rapid transit in the third month? (3)

(c) What proportion of people would be using the rapid transit in the long run (3)

5. Consider 4 light bulbs that have independent lifetimes exponentially distributed with mean 1 year. Suppose all the light bulbs are switched on at time $t = 0$ and let $X(t)$ denote the number of light bulbs that are still active at time t. Further assume that no two light bulbs would fail at the same time.

(a) Show that $X(t)$ is a continuous time Markov chain and find the rate matrix. (5)

(b) Find the average time the chain spends in each state. (2)

(c) What is the average time until the last bulb dies ? (2)

6. (a) The generator matrix of a continuous time Markov chain is (4)

$$P = \begin{bmatrix} -4 & 2 & 2 \\ 3 & -4 & 1 \\ 5 & 0 & -5 \end{bmatrix}$$

Find the steady sate distribution.

(b) The transition probability matrix of a discrete time Markov chain is (5)

$$\begin{bmatrix} 0.4 & 0.3 & 0.3 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 \\ 0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 \\ 0 & 0.3 & 0 & 0.3 & 0.4 \end{bmatrix}$$

Identify the transient and recurrent states, periodic and aperiodic states and the irreducible closed sets in the Markov chain. Also check whether the Markov chain is ergodic.

PART-C

(Answer any **two** questions. Each question carries **12 marks**.)

7. The arrival of calls at a switchboard is modeled as a Poisson process with rate of calls 0.1 per minute.

(a) What is the probability that the number of calls in a 5 minute interval is less than 3 ? (4)

(b) What is the probability that one call arrives during the first 10-minute interval and two calls arrive during next 10-minute interval ? (4)

(c) What is the probability that the 10-th arrival occurs 2 minutes after the 9-th arrival ? (4)

8. Suppose a person owns one share of stock currently worth Rs. 102. Assume that the value of this share over time changes according to

$$X(t) = 102 + B(t)$$

where $B(t)$follows a standard Brownian motion process where time is measured in months.

(a) What is the probability that the price 3 months from now is greater than 105 (6)

(b) Suppose that price of the stock hits 105 on the second month. What is the probability that the price will be 110 or greater in the third month ? (6)

9. (a) Arrivals of customers into a store follow a Poisson process with rate 20 per hour. Suppose that the probability that a customer buys something is 0.30. (6)

   (i) Find the expected number of sales made during an eight-hour business day.

   (ii) Find the probability that 10 or more sales are made in one hour.

   (b) Let $B(t)$ be the standard Brownian motion. Find (i) $P(B(2) \leq 3|B(1) = 1)$ and (ii) $P(B(1) + B(2) > 1)$. (6)

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