Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech S1 (Special Improvement) Examination January 2021 (2019 scheme)

Course Code: MAT101

Course Name: LINEAR ALGEBRA AND CALCULUS

(**2019-Scheme**)

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks.

1	Determine the rank of the matrix $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 3 & -1 \\ 5 & 3 & -2 \end{bmatrix}$	(3)
2	What kind of conic section is represented by the quadratic form $7x_1^{\ 2} + 6x_1 x_2 + 7x_2^{\ 2} = 200$ Transform it into canonical form.	(3)
3	Find the derivative of $w = x^2 + y^2$ with respect to t along the path $x = at^2,\ y = 2at$.	(3)
4	Let $f(x, y) = \sqrt{3x + 2y}$, find the slope of the surface $z = f(x, y)$ in the y -direction at the point (2, 5).	(3)
5	Evaluate $\int_0^a \int_0^a \int_0^a (yz + xz + xy)\,dx\,dy\,dz$	(3)
6	Use polar co-ordinates to evaluate $\int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} \left(x^2 + y^2\right)^{\frac{3}{2}} dy\,dx$	(3)
7	Test the convergence of $\sum_{k=1}^{\infty} \frac{1}{(2k+3)^{17}}$	(3)
8	Examine whether the series convergence or not $\sum_{k=1}^{\infty} \frac{1}{(\ln(k+1))^k}$	(3)
9	Find the Maclaurin series of $\frac{1}{x+1}$ up to third degree term.	(3)
10	Find the Fourier Half Range sine series of $f(x) = x$ in , $0 < x < \pi$.	(3)

PART B

Answer one full question from each module, each question carries 14 marks

Module-I

11 a) Test for consistency and solve the system of equations (7)

$$x + 2y - z = 3$$
$$3x - y + 2z = 1$$
$$2x - 2y + 3z = 2$$
$$x - y + z = -1$$

b) Find the eigenvalues and eigenvectors of $\begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$ (7)

12 a) For what values of a and b do the system of equations (7)

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + az = b$$

have i) no solution ii) unique solution iii) more than one solution.

b) Find the matrix of transformation that diagonalize the matrix (7)

$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. Also write the diagonal matrix.

Module-II

13 a) If If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$. (7)

b) If the local linear approximation of a function $f(x, y, z) = xy + z^2$ at a point P is $L(x, y, z) = y + 2z - x$, find the point P. (7)

14 a) If $z = e^{xy}$, $x = 2u + v$, $y = \frac{u}{v}$ find $\frac{\partial z}{\partial u}$. (7)

b) Locate all relative extrema of $f(x, y) = 3x^2 - 2xy + y^2 - 8y$. (7)

Module-III

15 a) Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dy\, dx$ by reversing the order of integration. (7)

b) Using triple integral find the volume of the solid in the first octant bounded by the coordinate planes and the plane $x + 2y + z = 6$. (7)

16 a) Find the mass and center of gravity of the triangular lamina with vertices (0,0), (0,1) and (1,0) and density function $\delta(x,y)=xy$ (7)

b) Evaluate $\iint_R x^2\,dy\,dx$, where R is the region between $y=x$ and $y=x^2$ (7)

Module-IV

17 a) Discuss the convergence of the series (7)

$$(i)\sum_{k=1}^{\infty}\frac{k!}{k^k} \qquad (ii)\sum_{k=1}^{\infty}\left(\frac{k}{k+1}\right)^{k^2}$$

b) Examine the convergence and divergence of the series (7)

$$\frac{x}{1.3}+\frac{x^2}{3.5}+\frac{x^3}{5.7}+\ldots\ldots$$

18 a) Test the convergence of $1+\frac{1.3}{3!}+\frac{1.3.5}{5!}+\frac{1.3.5.7}{7!}+\ldots\ldots\ldots$ (7)

b) Prove that the series $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{(k-2)}{k(k+1)}$ is conditionally convergent. (7)

Module-V

19 a) Obtain Fourier series for the function $f(x) = |\sin x| \quad -\pi<x<\pi$ (7)

b) If $f(x)=\begin{cases} kx\ ; & 0<x<\frac{\pi}{2} \\ k(\pi-x)\ ; & \frac{\pi}{2}<x<\pi \end{cases}$ then show that (7)

$$f(x)=\frac{4k}{\pi}\left(\frac{\sin x}{1^2}-\frac{\sin 3x}{3^2}+\frac{\sin 5x}{5^2}-\!-\!-\!-\right).$$

20 a) Find the Fourier cosine series of $f(x)=x^2$ in $(0,\pi)$. Hence show that (7)

$$1+\frac{1}{2^2}+\frac{1}{3^2}+---=\frac{\pi^2}{6}$$

b) Find the Fourier series for the function (7)

$$\begin{aligned} f(x) &= x & 0<x<1 \\ &= 1-x & 1<x<2 \end{aligned}$$

****