

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Third semester B.Tech examinations (S) September 2020

Course Code: CS201**Course Name: DISCRETE COMPUTATIONAL STRUCTURES**

Max. Marks: 100

Duration: 3 Hours

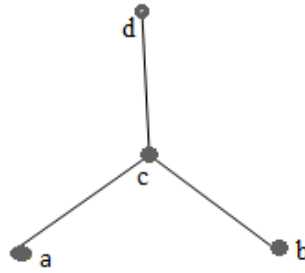
PART A*Answer all questions, each carries 3 marks.*

Marks

- 1 Show that $A \times (B \cap C) = (A \times B) \cap (A \times C)$ (3)
- 2 Prove that if a relation R on set A is transitive & irreflexive, then it is asymmetric. (3)
- 3 If 9 books are to be kept in 4 shelves, there must be atleast one shelf which contains atleast 3 books. Justify (3)
- 4 In how many ways can 6 boys and 4 girls sit in a row? In how many ways can they sit in a row if just the girls are to sit together? (3)

PART B*Answer any two full questions, each carries 9 marks.*

- 5 a) Consider f, g and h are functions on set of integers. (4)
 $f(n)=n+2$, $g(n)=n^2$, $h(n)=3n$. Determine fogoh, gofoh, hofog, fog.
- b) If $S=N \times N$ and the binary operation * is defined by $(a,b)*(c,d)=(ac,bd)$ for all a,b,c,d in N, show that $(S,*)$ is a semigroup. Is it monoid? (5)
- 6 a) Which of the following relations on $\{0,1,2,3\}$ are equivalence relation? Justify the answer. (4)
 $R1=\{(0,0),(1,1),(2,2),(3,3)\}$
 $R2=\{(0,0),(0,2),(2,0),(2,2),(2,3),(3,2),(3,3)\}$
 $R3=\{(0,0),(1,1),(1,2),(2,1),(2,2),(3,3)\}$
 $R4=\{(0,0),(1,1),(2,2),(3,3),(2,3),(3,2)\}$
- b) Find the number of integers between 1 and 250 both inclusive that are not divisible by any of the integers 2,3,5 (5)
- 7 a) Solve the recurrence relation $a_n - 2a_{n-1} = 3^n$; $a_1=5$ (5)
- b) Determine whether POSET represented by Hasse diagram given below have greatest element, least element, minimal element, maximal element. (4)



PART C

Answer all questions, each carries 3 marks.

- 8 Prove that the order of each sub group of a finite group G is a divisor of the order of group G . (3)
- 9 Let G be a group and suppose that a and b are any elements of G . Show if $(ab)^2 = a^2b^2$ then group is abelian. (3)
- 10 Let (L, \leq) be a lattice and a, b, c, d elements of L . (3)
Prove that if $a \leq c$ and $b \leq d$ then $a \vee b \leq c \vee d$
- 11 Define principle of duality in Boolean algebra. (3)

PART D

Answer any two full questions, each carries 9 marks.

- 12 a) Prove that the necessary and sufficient condition that a non-empty subset of a group G be a subgroup is $a, b \in H \Rightarrow ab^{-1} \in H$. (5)
- b) State and prove Absorption properties of lattice (4)
- 13 a) Is D_{12} a complemented lattice? Explain (4)
[D_{12} is set of divisors of 12 for the relation $R = \{(x, y) | x \text{ divides } y\}$]
- b) Show that $(Z, \bullet, *)$ is a ring where $a \bullet b = a + b - 1$ and $a * b = a + b - ab$ (5)
for every $a, b \in Z$ (Set of all integers).
- 14 a) Show that the set $\{1, 2, 3, 4, 5\}$ is not a group under addition modulo 6 (4)
- b) Define boolean algebra and explain how it is related to lattice. (5)

PART E

Answer any four full questions, each carries 10 marks.

- 15 a) Show that $(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)$ tautologically implies r without using truth table (5)
- b) Express the following statements in predicate logic. (5)
- i) Some students are clever.
- ii) All men are kind.
- iii) Some person in this class has visited the Grand Canyon.

- 16 a) Suppose x is a real number. Consider the statement “if $x^2=4$, then $x=2$ ” (5)
Construct the converse, inverse, contrapositive of the given statement
- b) Prove the following implication (5)
 $\forall x[P(x) \rightarrow Q(x)], \forall x[R(x) \rightarrow \neg Q(x)] \Rightarrow \forall x[R(x) \rightarrow \neg P(x)]$
- 17 a) Show that $p \rightarrow \neg s$ is a valid inference from premises $p \rightarrow (q \vee r)$, $q \rightarrow \neg p$, $s \rightarrow \neg r$ (5)
- b) Prove that $n(n+2)$ is divisible by 4 by mathematical induction, if n is any even positive integer. (5)
- 18 a) Show that $\neg p \wedge (\neg q \wedge r) \vee (q \wedge r) \vee (p \wedge r) \equiv r$ using truth table. (5)
- b) Given the premises (5)
P1: All men are selfish.
P2: All politicians are men.
Prove that the conclusion “All politicians are selfish” is a valid conclusion.
- 19 a) Show that $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$ is a tautology. (5)
- b) Prove that $\exists z Q(z)$ is a valid conclusion from the premises (5)
 $\forall x[P(x) \rightarrow Q(x)], \exists y P(y)$
- 20 a) If the product of two integers a and b is even, then either a is even or b is even. (5)
Prove the statement by contraposition.
- b) Using proof by contradiction method, prove that “if $3n+2$ is odd, then n is odd” (5)
