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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fourth semester B.Tech examinations (S), September 2020

Course Code: MA202

Course Name: PROBABILITY DISTRIBUTIONS, TRANSFORMS AND NUMERICAL METHODS

Max. Marks: 100 Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A (MODULES I AND II)

Answer two full questions.

1 a) Let Xbe a discrete random variable with mean 10 and variance 25. Find the positive values of α and β such that $Y = \alpha X - \beta$ has mean 0 and variance 1. 7

b) Derive the mean and variance of a Poisson Distribution. 8

2 a) If a continuous random variable has the probability distribution function 7

$$f(x) = \begin{cases} ke^{-3x} & if\ x > 0 \\ 0 & if\ x \leq 0 \end{cases}$$

then find (i) value of k (ii) $P[0 \leq X \leq 2]$ (iii) $P[X > 1.5\ ]$

b) In a Normal Distribution, if 6% of the items are below 60 and 39% are above 70, then find the mean and standard deviation. 8

3 a) Out of 2000 families with 4 children each, how many would you expect to have (i) at least one boy (ii) at most one boy 7

b) If X follows a uniform distribution in $(-2, 2)$, then (i) find $P[|X - 1| \leq 2]$ (ii) find k for which $P[X > k] = \frac{1}{3}$ (iii) Distribution function 8

PART B (MODULES III AND IV)

Answer two full questions.

4 a) Find the Fourier Sine Integral of $f(x) = \begin{cases} \sin x & if\ \ 0 \leq x \leq \pi \\ 0 & if\ \ x > \pi \end{cases}$ 7

b) Find the Fourier Cosine Transform of $f(x) = e^{-4x}$. Hence deduce that $\int_0^\infty \frac{\cos 2x}{x^2+16} dx = \frac{\pi}{8}\ e^{-8}$ 8

5 a) Using Convolution theorem, evaluate the Inverse Laplace Transform of $\frac{s}{(s^2+4)^2}$ 7

b) Evaluate (i) $L[t\ sin^2 2t]$ (ii) $L^{-1}\left[\frac{s+5}{s^2+4s+13}\right]$ 8

6 a) Find the Fourier Transform of $f(x) = \begin{cases} 1 & if\ |x| < 1 \\ 0 & if\ |x| > 1 \end{cases}$ Hence show that $\int_0^\infty \frac{\sin\omega}{\omega}\, d\omega = \frac{\pi}{2}$ 7

b) Solve using Laplace Transform: $y'' - 3y' + 2y = 4$ given $y(0) = 2, y'(0) = 3$ 8

PART C (MODULES V AND VI)

Answer two full questions.

7 a) Using Lagrange's interpolation formula, find a parabola of the form $y = ax^2 + bx + c$ passing through the points $(0,0), (2,4)$ and $(3,12)$ 6

b) Using Newton-Raphson Method, find the real root lying between 0 and 1 of $3x - \cos x - 1 = 0$. (Correct to three decimal places) 7

c) Apply Lagrange's interpolation formula to find y at $x = 2$ for the following values for $y = f(x)$. Given $f(0) = -12, f(1) = 0, f(3) = 6$ and $f(4) = 12$. 7

8 a) Solve by Gauss Elimination Method: 6

$3x + 4y + 5z = 18, \quad 2x - y + 8z = 13, \quad 5x - 2y + 7z = 20.$

b) Evaluate $I = \int_0^6 \frac{1}{1+x}\, dx$ using (i) Trapezoidal Rule (ii) Simpson's $\frac{1}{3}$ Rule (Take h=1). Also find the value of the integral by actual integration. 7

c) Using Euler's Method compute the value of $y(0.1)$ given $y' = x + \frac{1}{y}$, $y(0) = 1$ (Take $h = 0.025$) 7

9 a) Using Newton's Interpolation Formula find $f(1.2)$ and $f(2.0)$ from the table. 10

x	1	1.4	1.8	2.2
$y = f(x)$	3.49	4.82	5.96	6.50

b) Using Runge – Kutta Method of 4^{th} order, find $y(0.8)$ correct to four decimal places if $\frac{dy}{dx} = y - x^2$ given $y(0.6) = 1.7379$ (Take $h = 0.1$) 10