

Reg No.: \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
FOURTH SEMESTER B.TECH DEGREE EXAMINATION(S), DECEMBER 2019

**Course Code: ME202**

**Course Name: ADVANCED MECHANICS OF SOLIDS**

Max. Marks: 100

Duration: 3 Hours

**PART A**

*Answer any three questions, each carries 10 marks*

Marks

*Databook is not permitted to use in the exam*

- 1 a) Describe about the significance of stress invariants 2
- b) Given the stress matrix
- $$[\sigma] = \begin{bmatrix} 20 & 10 & 10 \\ 10 & 20 & 10 \\ 10 & 10 & 20 \end{bmatrix} \text{ MPa}$$
- (a) Determine the principal stresses and the direction cosines associated with the maximum principal stress. 5
- (b) Determine the maximum shear stress. 1
- (c) Determine the octahedral stresses 2
- 2 a) Write down the strain – displacement relation in Cartesian Coordinate system 2
- b) The displacement field for a body is given by  $\mathbf{u} = [2y^2\mathbf{i} + 4yz\mathbf{j} + (3 + 35z^2)\mathbf{k}]10^{-2}$ . Determine the strain tensor. 3
- c) State the conditions under which the following is a possible system of strains
- $$\varepsilon_{xx} = a + b(x^2 + y^2) + x^4 + y^4,$$
- $$\varepsilon_{yy} = \alpha + \beta(x^2 + y^2) + x^4 + y^4, \gamma_{xy} = A + Bxy(x^2 + y^2 - c^2), \gamma_{yz} = 0,$$
- $$\gamma_{zx} = 0 \quad \text{and} \quad \varepsilon_{zz} = 0.$$
- 5
- 3 a) For steel the Young's Modulus is  $207 \times 10^6$  kPa and Rigidity Modulus is  $80 \times 10^6$  kPa. For the strain tensor at a point, determine the stress tensor
- $$[\varepsilon_{ij}] = \begin{bmatrix} 0.001 & 0 & -0.002 \\ 0 & -0.003 & 0.003 \\ -0.002 & 0.003 & 0 \end{bmatrix}$$
- 5
- b) State and explain Saint Venant's principle for end effects 3
- c) Write the constitutive relation for linear elastic isotropic material 2
- 4 a) Obtain the stress compatibility equation for plane stress problem in the presence of body force 5
- b) Show that  $\phi = x^4 y + 4x^2 y^3 - y^5$  is a valid stress function and compute the stress 5

tensor for this case assuming a state of plane strain with Poisson ratio  $\nu = 0.25$ .

### PART B

*Answer any three questions, each carries 10 marks*

- 5 a) Derive the compatibility equation in terms of stress function  $\phi$  for polar co-ordinate system 7
- b) Write the equilibrium equation in polar coordinate system 3
- 6 a) In a very thick cylinder with outer radius much larger than the inner radius  $a$ , subjected to an external pressure  $P_0$  and zero internal pressure, prove that the radial and tangential stress variations are  $\sigma_r = P_0(1 - \frac{a^2}{r^2})$  and  $\sigma_\theta = P_0(1 + \frac{a^2}{r^2})$  10
- 7 a) Obtain the strain energy in terms of material properties for a shaft subjected to a torque  $T$ . 5
- b) For a given stress tensor at a point on a steel object with  $E = 207 \times 10^6$  kPa and  $G = 80 \times 10^6$  kPa, determine the value of strain energy density. 5

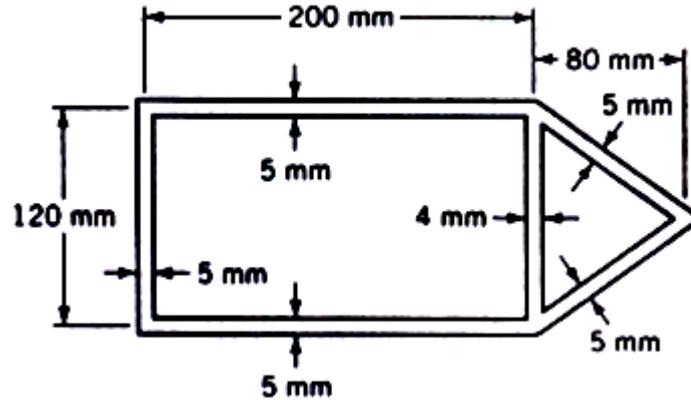
$$[\sigma] = \begin{bmatrix} 9 & 0 & 3 \\ 0 & -10 & 1 \\ 3 & 1 & 112 \end{bmatrix} \times 10^3 \text{ kPa}$$

- 8 a) Explain about unsymmetrical bending of beams. 2
- b) A beam symmetrical about  $y$ - $z$  axis is subjected to a bending moment  $M$ , about an arbitrary axis in the  $y$ - $z$  plane. Obtain the equation for flexural stress. 8

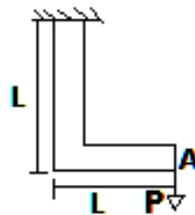
### PART C

*Answer any four questions, each carries 10 marks*

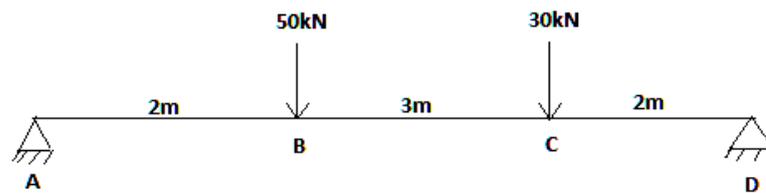
- 9 a) Verify that function  $\psi(x, y) = Axy$  is a Saint Venant's warping function, where  $A$  is a constant. Find the general expression for slope of tangent at a point on the boundary curve of the bar with this warping function. Find out shape of the cross section and  $J$  integral. 8
- b) Explain about centre of twist. 2
- 10 a) Derive the governing equation and boundary condition for torsion of non circular cross section in terms of Prandtl stress function. 8
- b) What is torsional rigidity? 2
- 11 a) The aluminium ( $G = 27$  GPa) hollow thin walled torsion member has dimensions as shown below. Its length is 3m. If the member is subjected torque of 11 kN-m, determine the maximum shear stress and angle of twist. 10



- 12 a) Based on the stress function approach, derive an expression for finding the maximum shear stress on a shaft of elliptical cross section? 10
- 13 a) A frame is subjected to a load  $P$  as shown in figure. The length of each leg is  $L$ . The frame has a constant flexural rigidity  $EI$ . If the effect of axial load is neglected, find the deflection at point  $A$  due to applied load  $P$ ? 6



- b) State and explain Castigliano's first and second theorem 4
- 14 a) 8



Using the principle of virtual work, find out the reactions at A and D

- b) Explain Maxwell theorem 2

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