

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
SECOND SEMESTER B.TECH DEGREE EXAMINATION(S), DECEMBER 2019

Course Code: MA102

Course Name: DIFFERENTIAL EQUATIONS

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks

- 1 Find a general solution of the ordinary differential equation $y'' + y = 0$ (3)
- 2 Reduce to first order and solve. $yy'' = 3(y')^2$. (3)
- 3 Find the particular integral of $y'' - 4y' - 5y = 4 \cos 2x$. (3)
- 4 Using a suitable transformation, convert the differential equation $(x^2 D^2 + xD + 1)y = \log x$ into a linear differential equation with constant coefficients. (3)
- 5 If $f(x)$ is a periodic function of period 2π defined in $[-\pi, \pi]$. Write down Euler's Formulas a_0, a_n, b_n for $f(x)$. (3)
- 6 Find the half range Fourier cosine series of the function $f(x) = x$ in the range $0 < x < 2$. (3)
- 7 Find the PDE by eliminating arbitrary function φ from $xyz = \varphi(x + y + z)$. (3)
- 8 Solve $(D + 2D')(D - 3D')^2 z = 0$. (3)
- 9 Write any three assumptions involved in the derivation of one dimensional wave Equation. (3)
- 10 A tightly stretched string of length l is fixed at both ends and pulled from its mid point to a height h and released from rest from this position. Write down the initial and boundary conditions. (3)
- 11 Write all possible solutions of one dimensional heat equation. (3)
- 12 Find the steady state temperature distribution in a rod of length l if the ends are kept at $0^\circ C$ and $100^\circ C$. (3)

PART B

Answer six questions, one full question from each module

Module 1

- 13 a) Solve $y'' - 2y' + y = 0, y(0) = 1, y'(0) = 2$. (6)
- b) Find a basis of solutions of the ODE $(x^2 - x)y'' - xy' + y = 0$, if $y_1 = x$ is a (5)

solution.

OR

14 a) Solve the ordinary differential equation $y'''' - 3y'' - 4y' + 6y = 0$. (6)

b) Solve the ordinary differential equation $xy'' + 2y' + xy = 0$, given that $y_1 = \frac{\sin x}{x}$ is a solution. (5)

Module 1I

15 a) By the method of variation of parameters, solve $y'' + 4y = \tan 2x$. (6)

b) Solve $y'' + 2y = x^2 e^{3x}$. (5)

OR

16 a) Solve $(x + 3)^2 y'' - 4(x + 3)y' + 6y = 3x$. (6)

b) Solve $x^2 y'' - 4xy' + 6y = x^5$. (5)

Module 1II

17 a) Find the Fourier series of f defined by $f(x) = x - x^2$ in $(-1, 1)$. (6)

b) Expand $f(x) = c$ in the half range sine-series in $0 \leq x \leq \pi$. (5)

OR

18 Obtain Fourier series for the function $f(x) = |\cos x|$, $-\pi \leq x \leq \pi$. (11)

Module 1V

19 a) Solve $r + s + 2t = e^{x+y}$. (6)

b) Find the general solution of $x^2(y - z)p + y^2(z - x)q = (x - y)z^2$. (5)

OR

20 a) Solve $(D^3 + D^2 D' - D D'^2 - D'^3)z = e^x \cos 2y$ (6)

b) Solve $(D^2 + 3DD' + 2D'^2)z = x^2 y^2$ (5)

Module V

21 A uniform elastic string of length 60 cm is subjected to a constant tension of 2 Kg. If the ends are fixed, the initial displacement $u(x, 0) = 60x - x^2$, $0 < x < 60$ and the initial velocity is zero, find the displacement function $u(x, t)$ (10)

OR

- 22 Find the deflection of the vibrating string which is fixed at the ends $x = 0$ and $x = 2$ and the motion is started by displacing the string into the form $\sin^3\left(\frac{\pi x}{2}\right)$ (10) and released it with zero initial velocity at $t = 0$.

Module VI

- 23 Find the temperature distribution in a rod of length $2m$ whose endpoints are maintained at temperature zero and initial temperature is $f(x) = 100(2x - x^2)$. (10)

OR

- 24 A rod of length 30cm has its ends A and B kept at $20^{\circ}C$ and $80^{\circ}C$ respectively until steady state temperature prevails. Suddenly the temperature at A is raised to $60^{\circ}C$ and the end B is decreased to $40^{\circ}C$. Find the temperature distribution in the rod at time t . (10)
